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A New Stability Analysis for Discrete-Time Switched Time-Delay Systems

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***Abstract*— The stability analysis problem is studied in this paper for a class of discrete-time switched time-delay systems described by delayed recurrence equations or a state matrix transformed after under the arrow form, by applying the comparison, the aggregations techniques and the M-matrix properties, new asymptotic stability conditions of such systems are derived under arbitrary switching. An illustrative example is presented permitting to understand the application of the proposed methods.**

***Keywords*— Discrete-time switched time-delay systems, Global asymptotic stability, Vector norms, M-matrix, Arrow matrix, Arbitrary switching**.

I. INTRODUCION

Switched systems belong to a special class of hybrid control systems, which consist of a family of continuous time or discrete time subsystems and a switching law that orchestrates the switching between them. Switched systems have strong engineering background in various areas and are often used as a unified modeling tool for a great number of real-world systems such as power electronics, chemical processes, mechanical systems, automotive industry, aircraft and air traffic control and many other fields. Therefore, switched systems have received extensive attention from many researchers and various results are available [1-6].

Time delay is the inherent feature of many physical processes, which may degrade system performance and lead to instability. In fact switched time-delay systems are represented by models witched depend on the courant states as well on the past ones.

The stability analysis present fundamental problems in the study of switched time delay systems in fact many methods are employed which has attracted growing attention in the literature [7-14]. In fact the lyapunov theory plays an important role. In this context our main aim consists on finding a larger stability domain and in the sense of various methods this paper intends to present new stability conditions for a particular class of discrete-time switched time-delay systems, by using the comparison, the aggregations techniques [15-22] and the M-matrix proprieties[23,24].

This particular class of the discrete-time switched time-delay systems considered in our work is given by subsystems represented by the following delayed recurrence equation:

$$y(k+n)+\left(\sum_{j=0}^{n-1} a_i^{n-j} y(k+j)+\sum_{j=0}^{n-1} d_i^{n-j} y(k+j-\tau)\right)=0 \quad (1)$$

when $i=1,\ldots,N$ represented the subsystems.

This paper is setup as follows: in the next section, we present the description and the problem formulation of the studied switched systems. In section III, sufficient stability conditions of these discrete-time switched time-delay systems based on the M-matrix properties are presented; next, we show the efficiency of the stability conditions given by application to time delay switched systems of the form (1). In section V, a validation on examples is drawn to show the effectiveness of the proposed, and finally, some concluding remarks are summarized in section VI.

II. PROBLEM FORMULATION AND PRELIMINAIES

In this section, we first give a mathematical description and notation:

Let $\Re^n$ denoted an n- dimensional linear vector space over the reals with the norm $\|.\|$. For any $u=(u_i)_{1\le i\le n}$, $v=(v_i)_{1\le i\le n}\in\Re^n$ we define the scalar product of the vector u and v as: $\langle u,v\rangle=\sum_{i=1}^{n} u_i v_i$.

Consider the following discrete-time switched systems with time delay formed by N subsystems described by the following state equation:

$$\begin{cases} x(k+1)=\sum_{i=1}^{N}\zeta_i(k)\left(A_i x(k)+D_i x(k-\tau)\right) \\ x(s)=\phi(s) \quad s=-\tau,-\tau+1,\ldots,-1 \end{cases} \quad (2)$$

where $x(k)\in\Re^n$ is the state vector of the system at time k, τ is the time delay of state, $x(s)$ denote the initial states vector, $A_i\,(i=1,...,N)$ and $D_i\,(i=1...N)$ are matrices of appropriate dimensions denoting the subsystems, and $N\geq 1$ denotes the number of subsystems.

The switching function ζ_i is an exogenous function which depends only on the time and not on the state, it is defined through:

$$\zeta_i(k)=\begin{cases}1 & \text{if } A_i \text{ and } D_i \text{ are active}\\ 0 & \text{otherwise}\end{cases} \quad \text{and} \quad \sum_{i=1}^{N}\zeta_i(k)=1 \tag{3}$$

III. MAIN RESULT

The following theorem given stability conditions for of discrete-time switched time-delay systems presented by (2).

Theorem 1. The discrete-time switched time-delay system given by (2) is asymptotically stable under arbitrary switching if the matrix T_c given by:

$$T_c=\left(|A_c|+|D_c|-I_n\right) \tag{4}$$

is the opposite of a $M-$matrix.

where:

$$A_c=\begin{bmatrix}\max\limits_{1\le i\le N}\left(a_i^{11}\right) & \cdots & \cdots & \max\limits_{1\le i\le N}\left(a_i^{1n}\right)\\ \vdots & \vdots & \vdots & \vdots\\ \vdots & \vdots & \vdots & \vdots\\ \max\limits_{1\le i\le N}\left(a_i^{n1}\right) & \cdots & \cdots & \max\limits_{1\le i\le N}\left(a_i^{nn}\right)\end{bmatrix} \tag{5}$$

and :

$$D_c=\begin{bmatrix}\max\limits_{1\le i\le N}\left(d_i^{11}\right) & \cdots & \cdots & \max\limits_{1\le i\le N}\left(d_i^{1n}\right)\\ \vdots & \vdots & \vdots & \vdots\\ \vdots & \vdots & \vdots & \vdots\\ \max\limits_{1\le i\le N}\left(d_i^{n1}\right) & \cdots & \cdots & \max\limits_{1\le i\le N}\left(d_i^{nn}\right)\end{bmatrix} \tag{6}$$

Proof: of any switched signal given by (3), let $\left(w_l>0,\forall l=1,...,n\right)$ and we consider the Lyapunov function defined such as:

$$v(k)=v_0(k)+\sum_{j=1}^{r}v_j(k) \tag{7}$$

where: $\begin{cases}v_0(k)=\left\langle |x(k)|,w\right\rangle\\ v_j(k)=\left\langle |D_i||x(k-j)|,w\right\rangle \quad \forall j\in 1,...,r\end{cases}$

demonstrated that:

$$\Delta v\left(x(k)\right)<\left\langle \left(|A_i|+|D_i|-I_n\right)|x(k)|,w\right\rangle \quad \forall r>0 \tag{8}$$

where :

$$\Delta v(k)=\Delta v_0(k)+\sum_{j=1}^{r}\Delta v_j(k) \tag{9}$$

and:

$$\begin{cases}\Delta v_0=\left\langle |x(k+1)|,w\right\rangle-\left\langle |x(k)|,w\right\rangle\\ \Delta v_j=\left\langle |D_i||x(k-j+1)|,w\right\rangle-\left\langle |D_i||x(k-j)|,w\right\rangle,\ j=1,...,r\end{cases}$$

knowing that:

$$\begin{aligned}&\left\langle |x(k+1)|,w\right\rangle=\left\langle |A_ix(k)+D_ix(k-r)|,w\right\rangle\\ &<\left\langle |A_i||x(k)|+|D_i||x(k-r)|,w\right\rangle\\ &=\left\langle |A_i||x(k)|,w\right\rangle+\left\langle |D_i||x(k-r)|,w\right\rangle\end{aligned}$$

next we have:

$$\sum_{j=1}^{r}\Delta v_j(k)=\Delta v_1(k)+\Delta v_2(k)+....+\Delta v_{r-1}(k)+\Delta v_r(k) \tag{10}$$

$$\begin{aligned}&=\left(\left\langle |D_i||x(k)|,w\right\rangle-\left\langle |D_i||x(k-1)|,w\right\rangle\right)\\ &+\left(\left\langle |D_i||x(k-1)|,w\right\rangle-\left\langle |D_i||x(k-2)|,w\right\rangle\right)\\ &+...\\ &+\left(\left\langle |D_i||x(k-r+1)|,w\right\rangle-\left\langle |D_i||x(k-r)|,w\right\rangle\right)\\ &=\left\langle |D_i||x(k)|,w\right\rangle-\left\langle |D_i||x(k-r)|,w\right\rangle\end{aligned}$$

them:

$$\Delta v(k)=\Delta v_0+\left\langle |D_i||x(k)|,w\right\rangle-\left\langle |D_i||x(k-r)|,w\right\rangle \tag{11}$$

$$\begin{cases}\Delta v_0=\left\langle |x(k+1)|,w\right\rangle-\left\langle |x(k)|,w\right\rangle\\ \left\langle |x(k+1)|,w\right\rangle<\left\langle |A_i||x(k)|,w\right\rangle+\left\langle |D_i||x(k-r)|,w\right\rangle\end{cases}$$

$$\begin{aligned}&\Delta v(k)<\left\langle |A_i||x(k)|,w\right\rangle+\left\langle |D_i||x(k-r)|,w\right\rangle-\left\langle |x(k)|,w\right\rangle\\ &+\left\langle |D_i||x(k)|,w\right\rangle-\left\langle |D_i||x(k-r)|,w\right\rangle\end{aligned}$$

and finally:

$$\begin{aligned}&\Delta v(k)<\left\langle |A_i||x(k)|,w\right\rangle-\left\langle |x(k)|,w\right\rangle+\left\langle |D_i||x(k)|,w\right\rangle\\ &=\left\langle \left(|A_i|+|D_i|-I_n\right)|x(k)|,w\right\rangle\left\langle \left(|A_c|+|D_c|-I_n\right)|x(k)|,w\right\rangle\end{aligned}$$

where the matrices A_c and D_c are defined in (5) and (6).

where $T_c=\left(|A_c|+|D_c|-I_n\right)$ defined by (4).

knowing that:

$$\left\langle \left(|A_c|+|D_c|-I_n\right)|x(k)|,w\right\rangle=\left\langle T_c|x(k)|,w\right\rangle \tag{12}$$

Now, suppose that T_c is the opposite of an matrix, according to the proprieties of the $M-$matrix, we can find a vector $\rho\in\Re_+^{*n}\left(\rho_l\in\Re_+^{*}\quad l=1,...,n\right)$ satisfying the relation $T_c^tw=-\rho,\forall w\in\Re_+^{*n}$, so we can write:

$$\left\langle T_c|x(k)|,w\right\rangle=\left\langle T_c^tw,|x(k)|\right\rangle=\left\langle -\rho,|x(k)|\right\rangle \tag{13}$$

Finally, we obtain:

$$\left\langle T_c \left|x(k)\right|, w\right\rangle = -\sum_{l=1}^{n} \rho_l \left|x_l(k)\right| < 0 \quad (14)$$

Then the discrete-time switched time-delay system given by (2) is asymptotically stable.

IV. APPLICATION TO DISCETE-TIME SWITCHED TIME-DELAY SYSTEMS DEFINED BY RECURRENCE EQUATION

In this part, as application is given for discrete-time switched time delay systems described by the following recurrence equation:

$$y(k+n) + \sum_{i=1}^{N} \zeta_i \left(\sum_{j=0}^{n-1} a_i^{n-j} y(k+j) + \sum_{j=0}^{n-1} d_i^{n-j} y(k+j-\tau) \right) = 0 \quad (15)$$

the switching functions ζ_i given by (3).

Then, the presence of delay-time terms makes the stability analysis for problem given by (15) difficult. Among solution, we will adopt the following change of variable:

$$x_{j+1}(k) = y(k+j) \quad j = 0, ..., n-1 \quad (16)$$

Equation (15) becomes:

$$\begin{cases} x_j(k+1) = x_{j+1}(k) \quad j = 1, ..., n-1 \\ x_n(k+1) = \sum_{i=1}^{N} \zeta_i \left(-\sum_{j=0}^{n-1} a_i^{n-j} x_{j+1}(k) - \sum_{j=0}^{n-1} d_i^{n-j} x_{j+1}(k-\tau) \right) \end{cases} \quad (17)$$

or under matrix representation:

$$x(k+1) = \sum_{i=1}^{N} \zeta_i \left(A_i x(k) + D_i x(k-\tau) \right) \quad (18)$$

where $x(k)$ is the state vector of components $x_j(k)$ $j = 0, ..., n-1$.

The matrices A_i and D_i, $i = 1, ..., N$ are defined such as:

$$A_i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 1 \\ -a_i^n & -a_i^{n-1} & \cdots & -a_i^i \end{bmatrix} \quad (19)$$

$$D_i = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ -d_i^n & -d_i^{n-1} & \cdots & -d_i^1 \end{bmatrix} \quad (20)$$

So, we define two associated polynomials of the subsystem (i) such as:

$$p_{A_i}(\lambda) = \lambda^n + \sum_{q=0}^{n-1} a_i^{n-q} \lambda^q \quad (21)$$

$$p_{D_i}(\lambda) = \sum_{q=0}^{n-1} d_i^{n-q} \lambda^q \quad (22)$$

In [17], a change of base for the system given by (5) under the arrow form gives:

$$z(k+1) = \sum_{i=1}^{N} \zeta_i \left(\tilde{A}_i z(k) + \tilde{D}_i z(k-\tau) \right) \quad (23)$$

where $z = Px$, $\tilde{A}_i$ and $\tilde{D}_i$ are matrices in the arrow form represented such as :

$$\tilde{A}_i = P^{-1} A_i P = \begin{bmatrix} \alpha_1 & 0 & \cdots & 0 & \beta_1 \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & \alpha_{n-1} & \beta_{n-1} \\ \gamma_i^1 & \cdots & \cdots & \gamma_i^{n-1} & \gamma_i^n \end{bmatrix} \quad (24)$$

$$\tilde{D}_i = P^{-1} D_i P = \begin{bmatrix} 0_{n-1,n-1} & 0_{n-1,1} \\ \delta_i^1 \cdots \delta_i^{n-1} & \delta_i^n \end{bmatrix} \quad (25)$$

and P is the corresponding passage matrix defined by:

$$P = \begin{bmatrix} 1 & 1 & \dots & 1 & 0 \\ \alpha_1 & \alpha_2 & \dots & \alpha_{n-1} & 0 \\ (\alpha_1)^2 & (\alpha_2)^2 & \dots & (\alpha_{n-1})^2 & \vdots \\ \vdots & \vdots & \dots & \vdots & 0 \\ (\alpha_1)^{n-1} & (\alpha_2)^{n-1} & \dots & (\alpha_{n-1})^{n-1} & 1 \end{bmatrix} \quad (26)$$

The elements of the matrix $\tilde{A}_i$ are defined by:

$$\beta_j = \prod_{\substack{q=1 \\ q \neq j}}^{n-1} \left(\alpha_j - \alpha_q \right)^{-1} \quad \forall\, j = 1, 2, ..., n-1$$

$$\begin{cases} \gamma_i^j = -P_{A_i}(\alpha_j) \quad \forall\, j = 1, 2, ..., n-1 \\ \gamma_i^n = -a_i^1 - \sum_{j=1}^{n-1} \alpha_j \end{cases} \quad (27)$$

and the elements of the matrix $\tilde{D}_i$ are:

$$\begin{aligned} \delta_i^j &= -p_{\tilde{D}_i}(\alpha_j) \quad j = 1, ..., n-1 \\ \delta_i^n &= -d_i^1 \end{aligned} \quad (28)$$

After this formulation, we can deduce the following theorem for the stability for a discrete-time switched time delay systems described by (15).

Theorem 2. The discrete-time switched time delay system described by (15) is globally asymptotically stable if there exist $\alpha_j \quad (j = 1,2,...,n-1)$, $\alpha_j \neq \alpha_q \ \forall j \neq q$, such as:

i) $$1 - |\alpha_j| > 0 \ \forall j = 1,2,...n-1 \quad (29)$$

ii) $$\left(\max_{1\le i\le N}\left(|\gamma_i^n|\right) + \max_{1\le i\le N}\left(|\delta_i^n|\right)\right) + \sum_{j=1}^{n-1}\left(\max_{1\le i\le N}\left(|\gamma_i^j|\right) + \max_{1\le i\le N}\left(|\delta_i^j|\right)\right)|\beta_j|\left(1-|\alpha_j|\right)^{-1} \in]0,1[\quad (30)$$

Prof: it suffices to verify that the matrix T_c is the opposite of an $M-$matrix .
Where:

$$T_c = |\tilde{A}_c| + |\tilde{D}_c| - I_n \quad (31)$$

Taking into account the previous matrix value; we obtain the matrix T_c as follows:

$$T_c = \begin{bmatrix} |\alpha_1|-1 & & & & |\beta_1| \\ & |\alpha_2|-1 & & & |\beta_2| \\ & & \ddots & & \vdots \\ & & & |\alpha_{n-1}|-1 & |\beta_{n-1}| \\ t_{c1} & t_{c2} & \cdots & t_{c(n-1)} & \begin{pmatrix}\max\limits_{\le i\le N}\left(|\gamma_n^i|\right) + \\ \max\limits_{\le i\le N}\left(|\delta_n^i|\right) - 1\end{pmatrix} \end{bmatrix} \quad (32)$$

Where, $t_{cj} = \max_{1\le i\le N}\left(|\gamma_i^j|\right) + \max_{1\le i\le N}\left(|\delta_i^j|\right) \ \forall j = 1,...,n-1$

Since the elements $\alpha_j \ \ j = 1,...,n-1$ can be arbitrarily selected, the choice $|\alpha_j| \in]0,1[$ with $\alpha_j \neq \alpha_q \ \ \forall j = 1,...,n-1$, permits to reduce checking the conditions of the opposite of an $M-$matrix to determining a simple test:

$$\begin{cases} |\alpha_j| \in]0,1[\quad j = 1,...,n-1 \\ \det(-T_c) > 0 \end{cases}$$

$$\det(-T_c) = (-1)^n \det(T_c) = \chi \prod_{j=1}^{n-1}\left(1-|\alpha_j|\right)$$

Where χ is given by:

$$\chi = 1 - \left(\max_{1\le i\le N}|\gamma_n^i| + \max_{1\le i\le N}\left(|\delta_n^i|\right)\right) - \sum_{j=1}^{n-1}\left(\max_{1\le i\le N}|\gamma_j^i| + \max_{1\le i\le N}\left(|\delta_j^i|\right)\right)|\beta_j|\left(1-|\alpha_j|\right)^{-1}$$

Them by applied the proprieties of $M-$matrix, we determine sufficient stability conditions for discrete-time switched time-delay system described by (15).

It comes $\chi > 0$. Them this condition is equivalent to:

$$\left(\max_{1\le i\le N}\left(|\gamma_n^i|\right) + \max_{1\le i\le N}\left(|\delta_n^i|\right)\right) + \sum_{j=1}^{n-1}\left(\max_{1\le i\le N}\left(|\gamma_j^i|\right) + \max_{1\le i\le N}\left(|\delta_j^i|\right)\right)|\beta_j|\left(1-|\alpha_j|\right)^{-1} \in]0,1[$$

V. ILLUSTRATIVE EXAMPLE

As an application of theorem 2, we consider the following example.

Example 1.

We consider the discrete-time switched time-delay system described by the following recurrence equation:

$$y(k+2) + \sum_{i=1}^{2}\zeta_i\left(\sum_{j=0}^{1}a_i^{2-j}y(k+j) + \sum_{j=0}^{1}d_i^{2-j}y(k+j-1)\right) = 0$$

So, the subsystems are defined by:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -0.01 & -0.02 \end{bmatrix}, \qquad D_1 = \begin{bmatrix} 0 & 1 \\ -0.01 & -0.6 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ -0.07 & -0.01 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} 0 & 1 \\ -0.05 & -0.04 \end{bmatrix}$$

The time-delay $\tau = 1$, according to (18), we get the following state representation:

$$x(k+1) = \sum_{i=1}^{N}\zeta_i\left(\begin{pmatrix} 0 & 1 \\ -a_i^2 & -a_i^1 \end{pmatrix}x(k) + \begin{pmatrix} 0 & 0 \\ -d_i^2 & -d_i^1 \end{pmatrix}x(k-1)\right)$$

After according to (23, 24, 25 and 26) change of base for the discrete-time switched time- delay system under the arrow form is given:

$$z(k+1) = \sum_{i=1}^{N}\zeta_i\left(\begin{pmatrix} \alpha & 1 \\ -p_{A_i}(\alpha) & -a_i^1-\alpha \end{pmatrix}z(k) + \begin{pmatrix} 0 & 0 \\ -p_{D_i}(\alpha) & -d_i^1 \end{pmatrix}z(k-1)\right)$$

Then, the stability conditions deduced from theorem 1 are:

i) $|\alpha| < 1$

ii) $$0 < \max\left(|\gamma_1^2|, |\gamma_2^2|\right) + \max\left(|\delta_1^2|, |\delta_2^2|\right) + \left(\max\left(|\gamma_1^1|, |\gamma_2^1|\right) + \max\left(|\delta_1^2|, |\delta_2^2|\right)\right)\left(1-|\alpha|\right)^{-1} < 1$$

We take $\alpha = 0.05$, condition (ii) is verified such as:

$$\max\left(|-0.07|, |-0.06|\right) + \max\left(|-0.04|, |-0.6|\right)$$
$$\left(\max\left(|-0.0135|, |-0.073|\right) + \max\left(|-0.04|, |-0.052|\right)\right)(1.052)$$
$$= 0.07 + 0.6 + (0.073 + 0.052)(1.052) = 0.8015 < 1$$

When we fixed the sampling time $T_e = 0.2s$, $t_f = kT_e = 10s$ the switched time $t_1 = k_1T_e = 5s$ and the original state vector $x(0) = [-1 \ \ -7]^T$, and $x_{-1} = [1 \ \ 0.5]^T$ the evolution of the states and the state space are given respectively by figure 1 and figure 2.

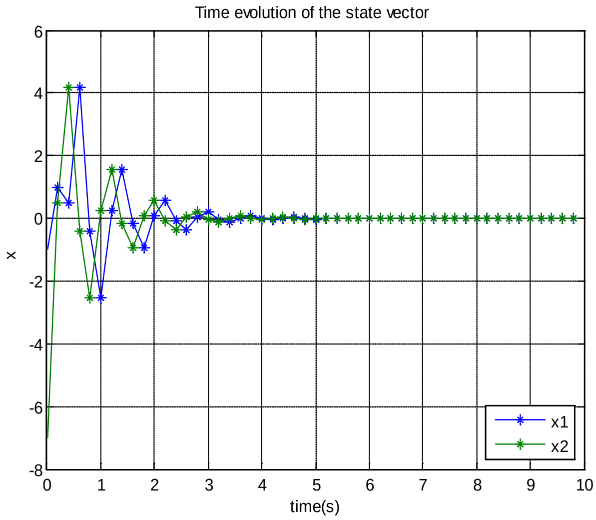

Figure 1. Time evolution of the state vector for example 1

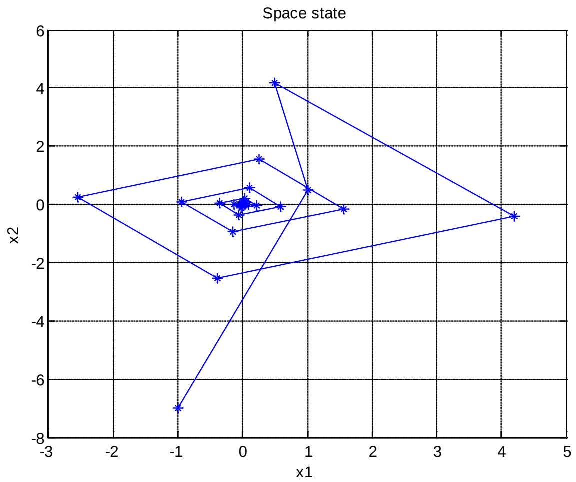

Figure 2. Space state for example 1

VI. CONCULSION

This paper has investigated a new stability conditions for discrete-time switched time-delay systems under arbitrary switching. These conditions were deduced from the application of the comparison, the aggregations techniques and the M-matrix proprieties. The main benefit of this technique that it avoids the problem of existence of Lyapunov functions. Finally, the effectiveness of the proposed method is illustrated by a numerical example.

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